RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2016-19]
B.A./B.Sc. SECOND SEMESTER (January – June) 2017
Mid-Semester Examination, March 2017

Date : 18/03/2017 **MATHEMATICS (General)**
Time : 12 noon – 1 pm Paper : II Full Marks : 25

Answer <u>any one</u> question from <u>Question nos. 1 & 2</u> : [1×5]

1. a) Let the original axes OX and OY be rotated through an angle θ in the anticlockwise direction such that OX' and OY' are the new set of axes. If (x, y) and (x', y') be the coordinates of the same point P referred to OX, OY and OX', OY' respectively. Then what is the relation between (x, y) and (x', y')? [1]

   b) Show that a+b, $ab-h^2$, f^2+g^2 obtained from $ax^2+2hxy+by^2+2gx+2fy+c$ remains invariant under transformation of rotation. [1+2+1]

2. Find the angle through which the axes are to be rotated so that the equation $x\sqrt{3}+y+6=0$ may be reduced to the form x = c. Also determine the value of c. [5]

Answer <u>any one</u> question from <u>Question nos. 3 & 4</u> : [1×6]

3. a) Find the vector equation of a line passing through two given points. [3]

   b) Find the equation to the line passing through the point (7, –3, 4) and parallel to the vector (2, –1, 1) and determine the point where it cuts the plane through the three points (2, 1, –3), (4, –1, 3), (3, 0, 1). [3]

4. Show that the internal bisector of any angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle. [6]

Answer <u>any one</u> question from <u>Question nos. 5 & 6</u> : [1×7]

5. a) Evaluate : $\lim\limits_{n\to\infty}\left(\sqrt[3]{n+1}-\sqrt[3]{n}\right)$ [3]

   b) Check if the series $\sum\limits_{n=1}^{\infty}\frac{1}{n}$ is convergent. [4]

6. a) Let $\{x_n\}$ and $\{y_n\}$ be real sequence such that $x_n \to 1$ and $y_n \to 2$. Show that $(x_n+y_n)\to 3$. [3]

   b) Show that a convergent real sequence is bounded. [4]

Answer <u>any one</u> question from <u>Question nos. 7 & 8</u> : [1×7]

7. a) Evaluate : $\lim\limits_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)...\left(1+\frac{n}{n}\right)\right\}^{1/n}$. [3]

   b) By Wallis' method, calculate $\int_a^b \frac{1}{x}dx, 0<a<b$. [4]

8. a) If $I_n = \int_0^{\pi/2} \cos^n x \, dx$ where n is a positive integer greater than 1, show that $I_n = \frac{n-1}{n} I_{n-2}$. Hence find $\int_0^1 x^2 \sqrt{1-x^2} \, dx$. [4]

b) Evaluate : $\int \frac{dx}{5+4\sin x}$. [3]

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